2017 Oral Exam: Probability and Statistics
Team

Problem 1. *An ant randomly travels along edges on a unit cube. Each step, it travels from one vertex to an adjacent vertex. Starting from a vertex, what is the expected number of steps the ant is to travel to first reach the diagonal vertex?*

Problem 2. *An algorithm called MM to maximize a function $g(x)$ is as follows. We find another easy-to-compute function $Q(x, z)$ such that*

$$g(z) \geq Q(x, z), \quad g(x) = Q(x, x).$$

Starting from an initial x_0, let $x_{k+1} = argmax_z Q(x_k, z)$. Argue that $g(x_k)$ is a nondecreasing sequence.

Let Y and X be two random vectors. Denote by $f_Y(\cdot;\theta), f_{X\,|\,Y}(\cdot,\cdot;\theta), f_{XY}(\cdot,\cdot;\theta)$ the marginal, conditional and joint densities that depend on parameter θ. We wish to maximize $l(\theta) := \log f_Y(y;\theta)$. Define

$$Q(\theta, \tilde{\theta}) = \int \log f_{XY}(x, y; \tilde{\theta}) f_{X|Y}(x, y; \theta) dx$$

Consider

$$\theta_{k+1} = argmax_{\tilde{\theta}}\ Q(\theta_k, \tilde{\theta}).$$

Argue that this algorithm, called EM, can be viewed as a special case of MM. As a result $l(\theta_{k+1}) \geq l(\theta_k)$.